

11.1 - Sequences

Preparation (Watch at Home)

11.1a

Exercise:

Calculate, to four decimal places, the first ten terms of the sequence and use them to plot the graph of the sequence by hand. Does the sequence appear to have a limit? If so, calculate it. If not, explain why.

$$a_n = 2 + \frac{(-1)^n}{n}$$

Theorem (statement):

If $\lim_{n \rightarrow \infty} |a_n| = 0$, then $\lim_{n \rightarrow \infty} a_n = 0$.

Definition: A sequence $\{a_n\}$ has the **limit** L and we write $\lim_{n \rightarrow \infty} = L$ or $a_n \rightarrow L$ as $n \rightarrow \infty$ if for every $\varepsilon > 0$ there is a corresponding integer N such that if $n > N$, then $|a_n - L| < \varepsilon$.

Exercise:

$$a_n = 1 + \frac{10^n}{9^n}$$

11.1b

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{3 + 5n^2}{1 + n}$$

$$a_n = \frac{4^n}{1 + 9^n}$$

$$a_n = e^{\frac{2n}{n+2}}$$

Theorems (statement):

If $\lim_{x \rightarrow \infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim_{x \rightarrow \infty} a_n = L$.

If $\lim_{x \rightarrow \infty} a_n = L$ and the function f is continuous at L , then $\lim_{x \rightarrow \infty} f(a_n) = f(L)$.

11.1d

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

$$a_n = \frac{1 - n}{2 + n}$$

Theorem (statement):

Monotonic Sequence Theorem: Every bounded, monotonic sequence is convergent.

In-Class Discussion

11.1c

Exercises:

Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\left\{ \frac{\ln n}{\ln(2n)} \right\}$$

$$a_n = \frac{(-3)^n}{n!}$$

11.1e

Exercise:

Show that the sequence defined by

$$a_1 = 2, a_{n+1} = \frac{1}{3 - a_n}$$

satisfies $0 \leq a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.



In-Class Practice

Determine whether the sequence converges or diverges. If it converges, find the limit.

1. $a_n = \cos\left(\frac{n\pi}{n+1}\right)$

2. $a_n = \frac{(-1)^{n+1}n}{n + \sqrt{n}}$

3. $a_n = \ln(n + 1) - \ln n$

Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

4. $a_n = 2 + \frac{(-1)^n}{n}$

a) By induction or otherwise, show that $\{a_n\}$ is increasing and bounded above by

3. Apply the Monotonic Sequence Theorem to show that $\lim_{n \rightarrow \infty} a_n$ exists.

[illegible]

