6.5 - Exponential Growth and Decay

Preparation (Watch at Home)

6.5a

Exponential growth and decay equation

$P(t) = P_0 e^{kt}$, where P_0 is the initial population and k is the constant relative growth rate

$$\frac{dP/dt}{P} = k \Rightarrow \frac{dP}{dt} = kP$$

$$\frac{dP}{P} = k\,dt$$

Exercise:

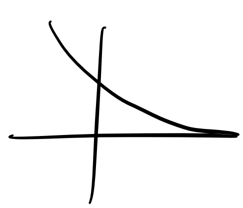
A bacteria culture grows with constant relative growth rate. The bacteria count was 400 after 2 hours and 25,600 after 6 hours.

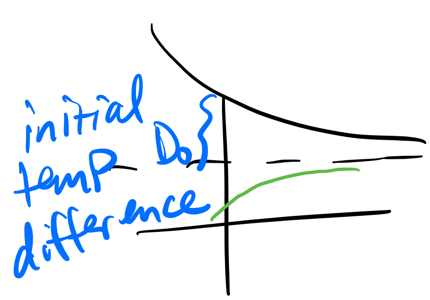
a) What is the relative growth rate? Express your answer as a percentage.
b) What was the initial size of the culture?
c) Find an expression for the number of bacteria after t hours.
d) Find the rate of growth after 4.5 hours.

$$\frac{dT}{dt} = k(T - T_s)$$

6.5b

Newton's Law of Cooling

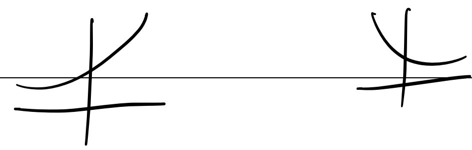




$T(t) = D_0 e^{kt} + T_s$, where D_0 is the initial temperature difference and T_s is the surrounding temperature

Recap: Constant relative growth rate: (decay — $k<0$)

$P(t) = P_0 e^{kt}$ $\quad k = 0.01$



In-Class Practice $\quad P(t) = P_0 e^{kt}$

1. A common inhabitant of the human intestines is the bacterium *Escherichia coli*, named after the German pediatrician Theodor Escherich, who identified it in 1885. A cell of this bacterium in a nutrient-rich broth medium divides into two cells every 20 minutes. The initial population of a culture is 50 cells. (P_0)

time to double

a) Find the relative growth rate. $\sim 208\%$

b) Find an expression for the number of cells after t hours. $P(t) = 50e^{2.08t}$

c) Find the number of cells after 6 hours.

d) Find the rate of growth after 6 hours. $P'(t) = 104e^{2.08t}$

$P'(6) = 104e^{2.08(6)}$

e) When will the population reach a million cells? $\approx 27{,}354{,}481$

$P_0 = 50$, time to double is 20 min $= \frac{1}{3}h$

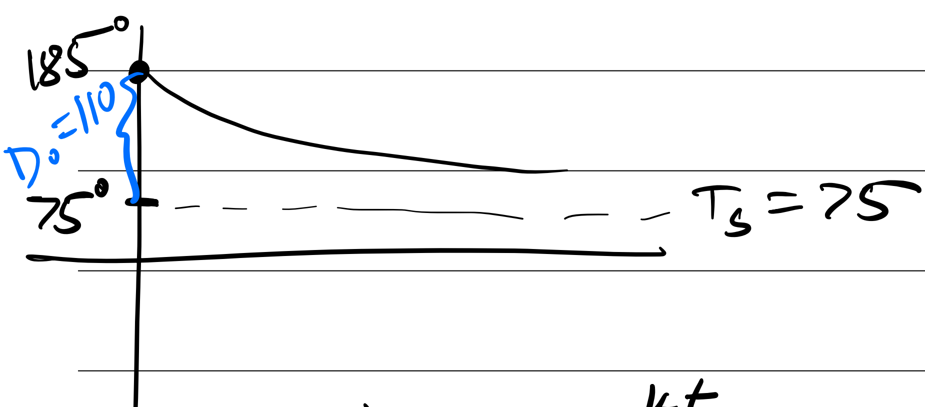
$P(\frac{1}{3}) = 100$

$$100 = 50e^{\frac{1}{3}k} \Rightarrow 2 = e^{\frac{1}{3}k}$$

$$k = 3\ln 2 \approx 2.079$$

$$P = 50e^{2.079t}$$

2. A roast turkey is removed from an oven when its temperature has reached 185° F and is placed on a table in a room where the ambient temperature is 75° F.
a) If the temperature of the turkey is 150° F after half an hour, what is the temperature after 45 minutes?
b) When will the turkey have cooled to 100° F?



$$T(t) = 110e^{kt} + 75 \qquad \underline{T(30) = 150 = 110e^{30k} + 75}$$

$$75 = 110e^{30k} \Rightarrow \frac{75}{110} = e^{30k}$$

$$\frac{1}{30}\ln\frac{75}{110} = k \approx -0.0128$$

$$T(t) = 110e^{-0.0128t} + 75$$

$$\frac{dT}{dt} = k(T - T_s) \Rightarrow \int \frac{dT}{T - T_s} = \int k\,dt$$

$$e^{\ln|T - T_s|} = e^{kt + C} = e^{kt} e^{C}$$

$$T - T_s = D_0 e^{kt}$$

initial temp difference

$$T(t) = T_s + D_0 e^{kt}$$