6.3 - Logarithmic Functions

Preparation (Watch at Home)

6.3b

Properties of logarithms

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_b MN = \log_b M + \log_b N$

$\log_b \frac{M}{N} = \log_b M - \log_b N$

$\log_b M^r = r \log_b M$

Change-of-base formula

$$\log_b x = \frac{\log_a x}{\log_a b}$$

In-Class Discussion

6.3a

Graphs and limits of logarithmic functions

Let $b = 2$

x	2^x
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

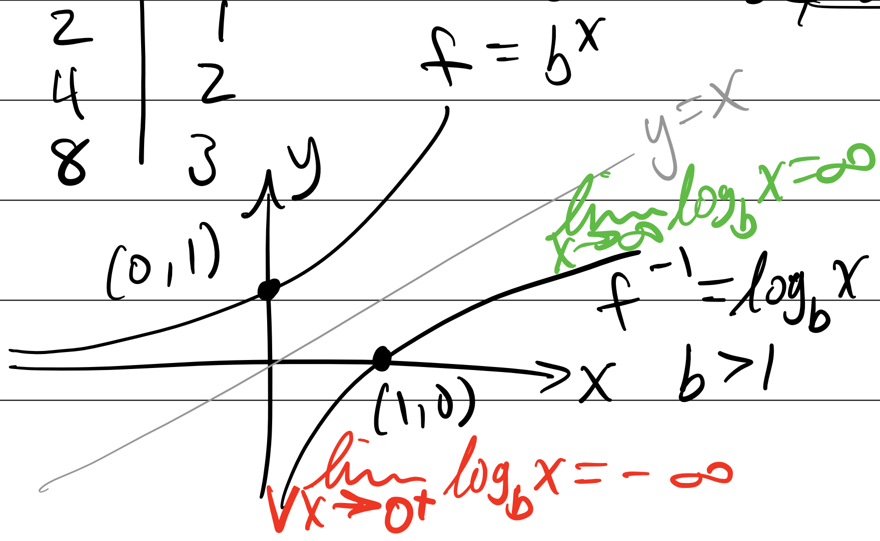
Consider $f(x) = b^x$, $b > 0$, $b \neq 1$

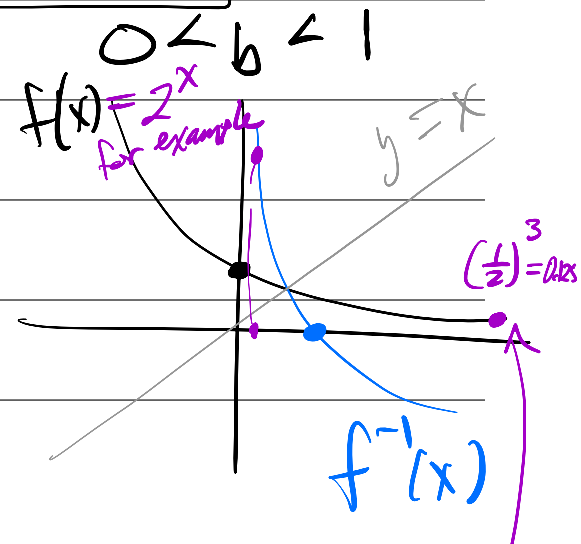
Then $f^{-1}(x) = \log_b x$ The exponent on the base b that gives x

1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

$f^{-1}(8) = \boxed{\log_2 8 = 3}$

because $\boxed{2^3 = 8}$





$(3, 0.125)$

$2^3 = 8$ is exponential form

$\log_2 8 = 3$ is logarithmic form

6.3c

Exercises:

Find the limit log base 5 of $8x - x^4$

$\lim_{x \to 2^-} \log_5 (8x - x^4)$

$5 > 1$

one approach: $x = 1.9, 1.99, 1.999$

$$\lim_{x \to 2^-} \log_5 [x(8 - x^3)] = \lim_{x \to 2^-} \log_5 x + \lim_{x \to 2^-} \log_5 (8 - x^3)$$

as $x \to 2^-$, $8 - x^3 \to 0^+$ so $-\infty$

$\lim_{x \to \infty} [\ln(2 + x) - \ln(1 + x)]$

$$\lim_{x \to \infty} \ln \frac{2+x}{1+x} = \ln \lim_{x \to \infty} \frac{2+x}{1+x}$$

$$\lim_{x \to \infty} \frac{2+x}{1+x} \cdot \frac{1/x}{1/x} = \lim_{x \to \infty} \frac{\cancel{\frac{2}{x}}^{\,0} + 1}{\cancel{\frac{1}{x}}^{\,0} + 1} = 1$$

$\ln 1 = 0$ because $e^0 = 1$

base is e

6.3d

Exercise:

On what interval is the curve $y = 2e^x - e^{-3x}$ concave downward?

In-Class Practice

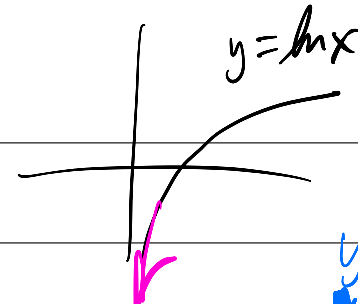
Use the properties of logarithms to expand the quantity.

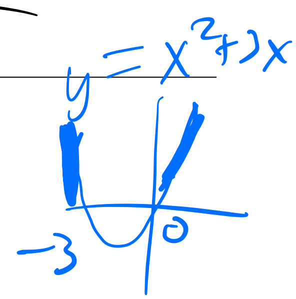
1. $\log_{10} \sqrt{\frac{x-1}{x+1}}$

2. $\ln\left(s^4\sqrt{t\sqrt{u}}\right)$

3. Find the limit: $\lim\limits_{x\to 0^+} \ln(\sin x)$ $= -\infty$

as $x \to 0^+$, $\sin x \to 0^+$





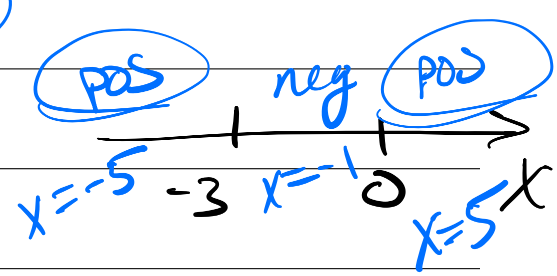
4. Find the domain of the function: $g(x) = \log_2 (x^2 + 3x)$

domain of $f(x) = \log_2 x$ is $(0, \infty)$

Here we need $x^2 + 3x > 0$

$x(x+3) > 0$

$0 \qquad -3$



Dom: $(-\infty, -3) \cup (0, \infty)$

5. Find the inverse function.

$$y = \frac{1 - e^{-x}}{1 + e^{-x}}$$

6. Find an equation of the tangent to the curve $y = e^{-x}$ that is perpendicular to the line $2x - y = 8$.