

5.5 - Average Value of a Function

5.5a

Average value of a function f over an interval $[a, b]$

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx$$

Exercise:

Find the average value of the function on the given interval.

$$g(t) = \frac{t}{\sqrt{3+t^2}}, [1, 3]$$

Theorem (statement):

The Mean Value Theorem for Integrals: If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$f(c) = f_{avg} = \frac{1}{b-a} \int_a^b f(x) dx \text{ that is, } \int_a^b f(x) dx = f(c)(b-a)$$

5.5b

Exercises:

- (a) Find the average value of f on the given interval.
- (b) Find c in the given interval such that $f_{avg} = f(c)$.
- (c) Sketch the graph of f and a rectangle whose base is the given interval and whose area is the same as the area under the graph of f .

$$f(x) = \sqrt[3]{x}, [0, 8]$$

Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.