

5.3 - Volumes by Cylindrical Shells

5.3a

Shell Method: The volume of a solid obtained by rotating the graph of a function f ,

$a \leq x \leq b$, $0 \leq a < b$ about the y -axis:

$$V = \int_a^b 2\pi x f(x) dx$$

Note that this is based on the surface area of a cylinder.

(An analogous formula exists for a function of y .)

Exercises:

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y -axis.

$$y = x^3, y = 0, x = 1, x = 2$$

5.3b

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the x -axis.

$$x + y = 4, x = y^2 - 4y + 4$$

When rotating a region between two curves f and g , $f \geq g$ for $a \leq x \leq b$ (about the y -axis), let $h(x) = f(x) - g(x)$ and we have

$$V = \int_a^b 2\pi x h(x) dx \text{ (An analogous formula exists for functions of } y \text{.)}$$

5.3c

When the axis of rotation is not a coordinate axis, we can generalize the volume as follows. If the axis of rotation is $x = c$, then let $R = |x - c|$. The above formula

becomes

$$V = \int_a^b 2\pi R h(x) dx$$

Exercise:

Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

$y = \sqrt{x}$, $x = 2y$; about $x = 5$