

5.2 - Volumes

5.2a

Disk Method: The volume of a solid obtained by rotating the graph of a function f ,

$a \leq x \leq b$ about the x -axis:

$$V = \int_a^b \pi [f(x)]^2 dx$$

Washer Method: The volume of a solid obtained by rotating the region between two functions, f and g , $a \leq x \leq b$ (for $f \geq g$ over the interval $[a, b]$) about the x -axis:

$$V = \int_a^b \pi \{[f(x)]^2 - [g(x)]^2\} dx$$

In both cases, the representative rectangle (represented as dx in these cases) is *perpendicular* to the axis of rotation. (Analogous formulas exist for functions of y .) Note that these are based on the area of a circle.

5.2b

Exercises:

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$$y = \frac{1}{x}, y = 0, x = 1, x = 4; \text{ rotate about the } x\text{-axis}$$

$$x = 2 - y^2, x = y^4; \text{ rotate about the } y\text{-axis}$$

5.2c

Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

$$y = x^3, y = 1, x = 2; \text{ rotate about } y = -3$$

When the axis of rotation is not a coordinate axis, we can generalize the volume of a solid of revolution as follows. If R is the distance between f and the axis of rotation and r is the distance between g and the axis of rotation, the above formulas become

$$V = \pi \int_a^b R^2 dx$$

$$V = \pi \int_a^b (R^2 - r^2) dx$$

Various possible disk and washer setups—focus on identifying R and r

5.2d

Definition: (volume—by slicing)

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x , through x and perpendicular to the x -axis, is $A(x)$, where A is a continuous function, then the volume of S is

$$V = \int_a^b A(x) dx$$

Exercises:

Find the volume of the described solid S .

The base of S is a circular disk with radius r . Parallel cross-sections perpendicular to the base are squares.

5.2e

The base of S is the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. Cross-sections perpendicular to the y -axis are equilateral triangles.