

5.1 - Areas Between Curves

5.1a

The area between curves

The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all x in $[a, b]$, is

$$A = \int_a^b [f(x) - g(x)] dx$$

An analogous expression exists for functions of y .

5.1b

Exercises:

Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y . Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

$$y = x^2, y = 4x - x^2$$

Sketch the region enclosed by the given curves and find its area.

$$x = y^4, y = \sqrt{2 - x}, y = 0$$

5.1c

In general, the area between f and g for $a \leq x \leq b$ is

$$A = \int_a^b |f(x) - g(x)| dx$$

Exercise:

Sketch the region enclosed by the given curves and find its area.

$$y = \cos x, y = 1 - \cos x, 0 \leq x \leq \pi$$