

4.4 - Indefinite Integrals and the Net Change Theorem

4.4a

Distinction between a definite integral and an indefinite integral

For $f(x) = F'(x)$:

Definite integral: $\int_a^b f(x) dx = F(b) - F(a)$, which is a *number*

Indefinite integral: $\int f(x) dx = F(x) + C$, which is a *function of x*

Exercises:

Find the general indefinite integral.

$$\int \left(u^2 + 1 + \frac{1}{u^2} \right) du$$

$$\int \sec t (\sec t + \tan t) dt$$

4.4b

Evaluate the definite integral.

$$\int_0^2 |2x - 1| dx$$

Theorem (statement):

Net Change Theorem: The integral of a rate of change is the net change:

$$\int_a^b F'(x) dx = F(b) - F(a)$$

Exercises:

A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t) dt$ represent?

4.4c

The velocity function (in m/s) is given for a particle moving along a line. Find (a) the displacement and (b) the distance traveled by the particle during the given time interval.

$$v(t) = t^2 - 2t - 3, 2 \leq t \leq 4$$