

4.3 - The Fundamental Theorem of Calculus

4.3a

Theorems (statement and proof):

The Fundamental Theorem of Calculus, Part 1: If f is continuous on $[a, b]$, then the function $g(x) = \int_a^x f(t) dt$, $a \leq x \leq b$ is continuous on $[a, b]$, and $g'(x) = f(x)$.

That is, $\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x)$ (differentiation is the inverse of integration).

4.3b

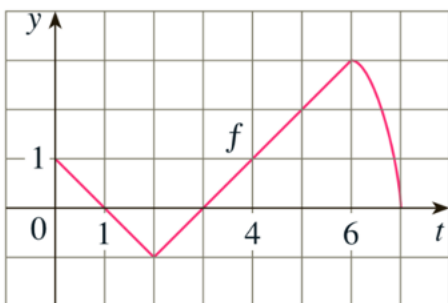
The Fundamental Theorem of Calculus, Part 2: If f is continuous on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$, where F is any antiderivative of f [that is, $F'(x) = f(x)$].

4.3c

Exercises:

Let $g(x) = \int_0^x f(t) dt$, where f is the function whose graph is shown.

- (a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6 .
- (b) Estimate $g(7)$.
- (c) Where does g have a maximum value? Where does it have a minimum value?
- (d) Sketch a rough graph of g .



4.3d

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$h(u) = \int_0^u \frac{\sqrt{t}}{t+1} dt$$

$$h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$$

Note: The Chain Rule applies to the derivative, as appropriate:

$$\frac{d}{dx} \left[\int_a^{u(x)} f(t) dt \right] = f(u(x))u'(x)$$

4.3e

Evaluate the integral.

$$\int_0^1 (1 - 8v^3 + 16v^7) dv$$

$$\int_0^4 (4-t)\sqrt{t} dt$$

$$\int_{\pi/4}^{\pi/3} \csc^2 \theta d\theta$$