

4.2 - The Definite Integral

4.2a

Definitions: (definite integral and integrable)

The definite integral of f from a to b is

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$
 where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$, provided the limit exists. If the limit does exist, we say that f is integrable on $[a, b]$.

Exercise:

Evaluate the definite integral.

$$\int_1^4 (x^2 - 4x + 2) dx$$

Summation formulas

$$\sum_{i=1}^n c = cn$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \left[\frac{n(n+1)}{2} \right]^2$$

4.2b

Exercise:

Express the limit as a definite integral on the given interval.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \sqrt{1 + x_i^3} \Delta x; [2, 5]$$

Properties of the definite integral

$$\int_a^b f(x) dx = -\int_b^a f(x) dx$$

$$\int_a^a f(x) dx = 0$$

$$\int_a^b c dx = c(b - a), \text{ where } c \text{ is any constant}$$

$$\int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^b c f(x) dx = c \int_a^b f(x) dx$$

$$\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx$$

Exercise:

Evaluate the integral by interpreting it in terms of areas.

$$\int_{-5}^5 \left(x - \sqrt{25 - x^2} \right) dx$$

4.2c

Comparison properties of the definite integral

If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq 0$ (which gives the area under the graph of f from a to b)

If $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$

If $m \leq f(x) \leq M$ for $a \leq x \leq b$, then $m(b - a) \leq \int_a^b f(x) dx \leq M(b - a)$

Exercises:

Use a comparison property of the definite integral to estimate the value of the integral.

$$\int_0^3 \frac{1}{x+4} dx$$