

4.1 - The Area and Distance Problems

4.1a

Introduction to the concept of areas and distances

4.1b

Exercise:

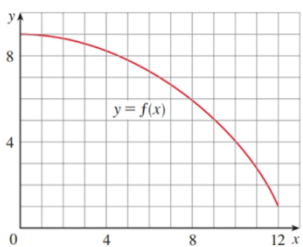
(a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.

- (i) L_6 (sample points are left endpoints)
- (ii) R_6 (sample points are right endpoints)
- (iii) M_6 (sample points are midpoints)

(b) Is L_6 an underestimate or overestimate of the true area?

(c) Is R_6 an underestimate or overestimate of the true area?

(d) Which of the numbers L_6 , R_6 , or M_6 gives the best estimate? Explain.



4.1c

Definition: (area)

The area A of the region S that lies under the graph of the continuous function f is the limit of the sum of the areas of approximating rectangles:

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} [f(x_1) \Delta x + f(x_2) \Delta x + \dots + f(x_n) \Delta x]$$

ALSO

$$A = \lim_{n \rightarrow \infty} L_n = \lim_{n \rightarrow \infty} [f(x_0) \Delta x + f(x_1) \Delta x + \dots + f(x_{n-1}) \Delta x],$$

where R_n indicates rectangles whose heights are given using the right endpoints of subintervals and L_n uses left endpoints of subintervals

Exercises:

Use the definition of area to express the area under f .

$$f(x) = x^2 + \sqrt{1 + 2x}, 4 \leq x \leq 7$$

4.1d

Determine a region whose area is equal to the given limit. Do not evaluate the limit.

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \sqrt{1 + \frac{3i}{n}}$$

The velocity graph of a car accelerating from rest to a speed of 120 km/h over a period of 30 seconds is shown. Estimate the distance traveled during this period.

