

3.4 - Limits at Infinity; Horizontal Asymptotes

3.4a

Infinite limits vs. limits at infinity

Definition: (limit at infinity)

Let f be a function defined on some interval (a, ∞) . Then $\lim_{x \rightarrow \infty} f(x) = L$ means that for every $\varepsilon > 0$ there is a corresponding number N such that if $x > N$ then $|f(x) - L| < \varepsilon$. (The definition for a limit at negative infinity is analogous.)

Definition: (horizontal asymptote)

The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either $\lim_{x \rightarrow \infty} f(x) = L$ or $\lim_{x \rightarrow -\infty} f(x) = L$.

Theorem (statement):

If $r > 0$ is a rational number, then $\lim_{x \rightarrow \infty} \frac{1}{x^r} = 0$. If $r > 0$ is a rational number such that x^r is defined for all x , then $\lim_{x \rightarrow -\infty} \frac{1}{x^r} = 0$.

Exercises:

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \frac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$$

$$\lim_{x \rightarrow -\infty} \frac{4x^3 + 6x^2 - 2}{2x^3 - 4x + 5}$$

3.4b

$$\lim_{x \rightarrow \infty} \left(\sqrt{4x^2 + 3x + 2x} \right)$$

$$\lim_{x \rightarrow -\infty} \left(\sqrt{4x^2 + 3x + 2x} \right)$$

3.4c

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 + 1}}{3x - 1}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 1}}{3x - 1}$$