

3.3 - What Derivatives Tell Us about the Shape of a Graph

3.3a

Exercise:

Analysis of the graph of $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Increasing/decreasing test

- (a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- (b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

The First Derivative Test

Suppose that c is a critical number of a continuous function f .

- (a) If f' changes from positive to negative at c , then f has a local maximum at c .
- (b) If f' changes from negative to positive at c , then f has a local minimum at c .
- (c) If f' is positive to the left and right of c , or negative to the left and right of c , then f has no local maximum or minimum at c .

3.3b

Definition: (concave upward/concave downward)

If the graph of f lies above all of its tangents on an interval I , then f is called concave upward on I . If the graph of f lies below all of its tangents on I , then f is called concave downward on I .

Concavity Test

- (a) If $f''(x) > 0$ on an interval I , then the graph of f is concave upward on I .
- (b) If $f''(x) < 0$ on an interval I , then the graph of f is concave downward on I .

Definition: (inflection point)

A point P on a curve $y = f(x)$ is called an inflection point if f is continuous there and the curve changes from concave upward to concave downward or from concave downward to concave upward at P .

The Second Derivative Test

Suppose f'' is continuous near c .

- (a) If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c .
- (b) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c .

3.3c

Exercise:

For the function $f(x) = 5x^{2/3} - 2x^{5/3}$, find the following.

- (a) Intervals on which f is increasing or decreasing.
- (b) Local maximum and minimum values of f .
- (c) Intervals of concavity and the inflection points.
- (d) Then use the information from parts (a)-(c) to sketch the graph.