

3.2 - The Mean Value Theorem

3.2a

Theorem (statement):

Rolle's Theorem: Let f be a function such that:

1. f is continuous on $[a, b]$.
2. f is differentiable on (a, b) .
3. $f(a) = f(b)$.

Then there is a number $c \in (a, b)$ such that $f'(c) = 0$.

Exercises:

Verify that the function satisfies the three hypotheses of Rolle's Theorem on the given interval. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

$$f(x) = x^3 - 2x^2 - 4x + 2, [-2, 2]$$

Let $f(x) = \tan x$. Show that $f(0) = f(\pi)$ but there is no number c in $(0, \pi)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

3.2b

Proof:

Rolle's Theorem

Theorem (statement and proof):

Mean Value Theorem: Let f be a function such that:

1. f is continuous on $[a, b]$.
2. f is differentiable on (a, b) .

Then there is a number $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Mathematical theory (how math works)

3.2c

Exercises:

Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{1}{x}, [1, 3]$$

Show that the equation has exactly one real root.

$$2x - 1 - \sin x = 0$$