

3.1 - Maximum and Minimum Values

3.1a

Definition: (local maximum)

A function f has a local maximum value at c if there is an open interval I containing c such that $f(c) \geq f(x)$ for all $x \in I$. The local maximum is $f(c)$. (The definition for local minimum is analogous.)

Definition: (absolute maximum)

A function f has an absolute maximum at c if $f(c) \geq f(x)$ for all x in the domain of f . The absolute maximum is $f(c)$. (The definition for absolute minimum is analogous.)

3.1b

Theorem (statement):

Extreme Value Theorem: If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

3.1c

Definition: (critical number)

The number c is a critical number of f if c is in the domain of f and $f'(c) = 0$ or $f'(c)$ does not exist.

3.1d

Exercise:

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = x^3 - 6x^2 + 5, [-3, 5]$$

Theorem (statement and proof):

Fermat's Theorem: If f has a local maximum or local minimum at c and if $f'(c)$ exists, then $f'(c) = 0$.