

2.5 - The Chain Rule

2.5a

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Alternative form of the Chain Rule (Leibnitz notation)

If $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Exercises:

Differentiate.

$$y = \sin(x^2)$$

2.5b

$$y = (\sin \sqrt{x})^3$$

$$f(t) = t \sin \pi t$$

$$y = \cos(ax)$$

2.5c

$$y = \sqrt{\sin(1 + x^2)}$$

$$y = x \sin \frac{1}{x}$$

$$y = \sin(t + \cos \sqrt{t})$$

2.5d

The General Power Rule

$$\frac{d}{dx} [f(x)^n] = n [f(x)]^{n-1} f'(x)$$

Exercises:

Differentiate.

$$F(x) = (1 + x + x^2)^{99}$$

$$U(y) = \left(\frac{y^4 + 1}{y^2 + 1} \right)^5$$

Proof:

The differentiation formula for the secant function