

2.2 - The Derivative as a Function

2.2a

The derivative of f as a function of x

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Exercise:

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$f(x) = \sqrt{x-6}$$

Notation for derivatives

$$f'(x) = \frac{d}{dx} [f(x)] = y' = \frac{dy}{dx} = D_x \text{ all represent the derivative of } f \text{ (or } y) \text{ with respect to } x$$

$$f'(2) = \left. \frac{dy}{dx} \right|_{x=2} \text{ represents the derivative of } f \text{ (or } y) \text{ evaluated at } x = 2$$

Higher-order derivatives

$$f''(x) = y'' = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d^2 y}{dx^2} \text{ represent a second derivative}$$

$f'''(x)$ is the third derivative, but for the fourth and higher, we use $f^{(4)}(x)$, etc.

Position, velocity, acceleration, and jerk

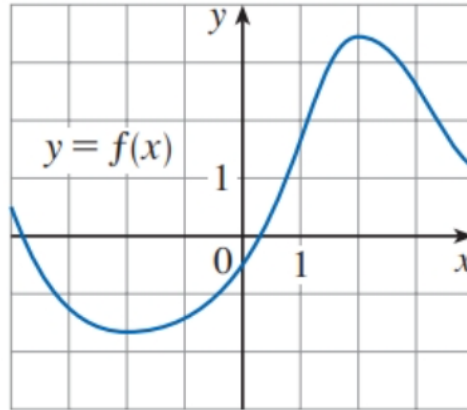
If $s(t)$ is a position function, then $v(t) = s'(t)$ is velocity, $a(t) = s''(t)$ is acceleration, and $j(t) = s'''(t)$ is the jerk.

2.2b

Exercises:

Use the given graph to estimate the value of each derivative. Then sketch the graph of f' .

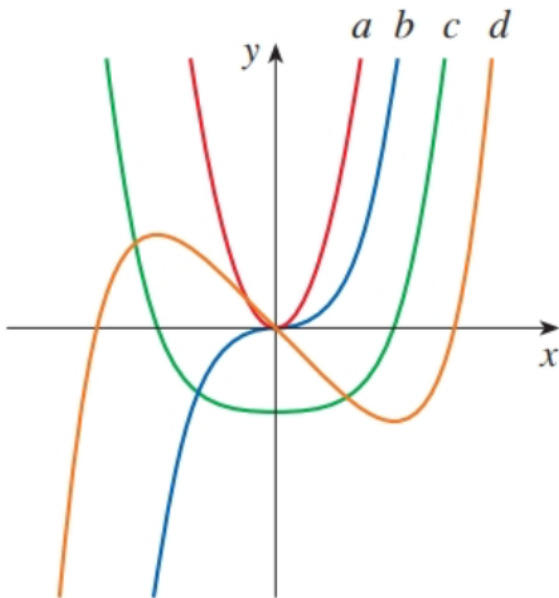
(a) $f'(-3)$ (b) $f'(-2)$ (c) $f'(-1)$ (d) $f'(0)$



(e) $f'(1)$ (f) $f'(2)$ (g) $f'(3)$

2.2c

The figure shows graphs of f , f' , f'' , and f''' . Identify each curve, and explain your choices.



2.2d

Definition: (differentiable)

A function f is differentiable at a if $f'(a)$ exists. It is differentiable on an open interval (a, b) [or (a, ∞) or $(-\infty, b)$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

Theorem (statement and proof):

If f is differentiable at a , then f is continuous at a .