

2.1 - Derivatives and Rates of Change

2.1a

Definition: (derivative)

The derivative of a function f at a is $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$, provided the limit exists.

Alternative form of the derivative of f at a

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

Equation of a tangent line at $(a, f(a))$

$$y - f(a) = f'(a)(x - a)$$

Exercises:

Find the equation of the tangent line to the curve at the given point.

$$y = x^3 - 3x + 1, (2, 3)$$

2.1b

$$y = \frac{2x + 1}{x + 2}, (1, 1)$$

The limit represents the derivative of some function f at some number a . Give the function f and number a .

$$\lim_{x \rightarrow 1/4} \frac{\frac{1}{x} - 4}{x - \frac{1}{4}}$$

Find $f'(a)$.

$$f(x) = \frac{4}{\sqrt{1-x}}$$

2.1c

The displacement (in feet) of a particle moving in a straight line is given by $s = \frac{1}{2}t^2 - 6t + 23$, where t is measured in seconds.

(a) Find the average velocity over each time interval:

(i) $[4, 8]$ (ii) $[6, 8]$

(iii) $[8, 10]$ (iv) $[8, 12]$

(b) Find the instantaneous velocity when $t = 8$.

(c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part (a). Then draw the tangent line whose slope is the instantaneous velocity in part (b).