

1.6 - Calculating Limits Using the Limit Laws

1.6a

Limit laws

Suppose that c is a constant and the limits $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. Then

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) \text{ (Sum/Difference Laws)}$$

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x) \text{ (Constant Multiple Law)}$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) \text{ (Product Law)}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ if } \lim_{x \rightarrow a} g(x) \neq 0 \text{ (Quotient Law)}$$

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n \text{ (Power Law)}$$

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)} \text{ (Root Law)}$$

[If n is even, we assume that $\lim_{x \rightarrow a} f(x) > 0$.]

$$\lim_{x \rightarrow a} c = c \text{ (Limit of a Constant)}$$

$$\lim_{x \rightarrow a} x = a \text{ (Identity Limit)}$$

Exercise:

Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{x \rightarrow -1} (x^4 - 3x)(x^2 + 5x + 3)$$

1.6b

Exercises:

Evaluate the limit, if it exists.

$$\lim_{x \rightarrow -1} \frac{2x^2 + 3x + 1}{x^2 - 2x - 3}$$

$$\lim_{u \rightarrow 2} \frac{\sqrt{4u + 1} - 3}{u - 2}$$

1.6c

$$\lim_{t \rightarrow 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right)$$

Find the limit, if it exists. If the limit does not exist, explain why.

$$\lim_{x \rightarrow -6} \frac{2x + 12}{|x + 6|}$$

1.6d

Theorem (statement):

The Squeeze Theorem: If $f(x) \leq g(x) \leq h(x)$ on some interval containing a (except possibly at a) and if $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$.

Exercise:

Find $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$.